

MÜHENDİSLİKTE DENEYSEL METODLAR

Grafik çizimi veya Bilgisayar hasaplaması ile olasılık yaklaşımı

On ölçümden elde edilen değerler

Reading	x_i , cm
1	4.62
2	4.69
3	4.86
4	4.53
5	4.60
6	4.65
7	4.59
8	4.70
9	4.58
10	4.63
$\sum x_i = 46.45$	

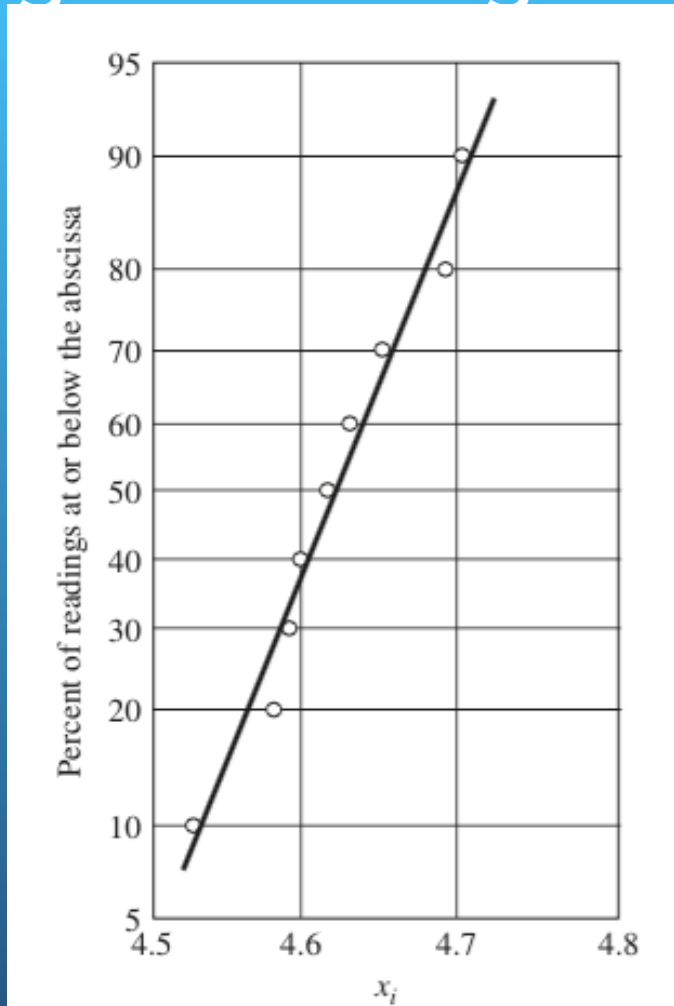
- Ortalama değeri hesaplanırsa;

$$x_m = \frac{1}{10} \sum x_i = \frac{1}{10} (46.45) = 4.645 \text{ cm}$$

- Standart sapma;

$$\sigma = \left[\frac{1}{n} \sum (x_i - x_m)^2 \right]^{1/2} = 0.0864$$

Değerlerden grafik çizilirse;



$$x = 4.62,$$

Çizilen yeni grafiğin ortalama değerinin 4.62 olduğu görülebilir.

Ortalam değer deney verilerine göre yapılan hesaplanmada 4.86 olduğu bilinmektedir.

Ancak değişimin bir doğru olduğu görülmektedir. Deney sonucu ile %100 örtüşmediği anlaşılmaktadır.

Bilgisayar Hesaplaması ile yaklaşım

The values of x are extended below and above the minimum and maximum value of 4.53 and 4.86 in order to pick up the tails of the normal distribution. Next, the actual cumulative frequencies are listed in the second column of the table. Note that there are no data points below $x = 4.53$ so these entries for the actual frequencies are zero. Likewise, all points have been observed at $x = 4.86$ or greater, so the actual frequencies are 1.0 at this point and above. The values for the normal distribution frequencies are computed with

$$\eta = \frac{x - x_m}{\sigma}$$

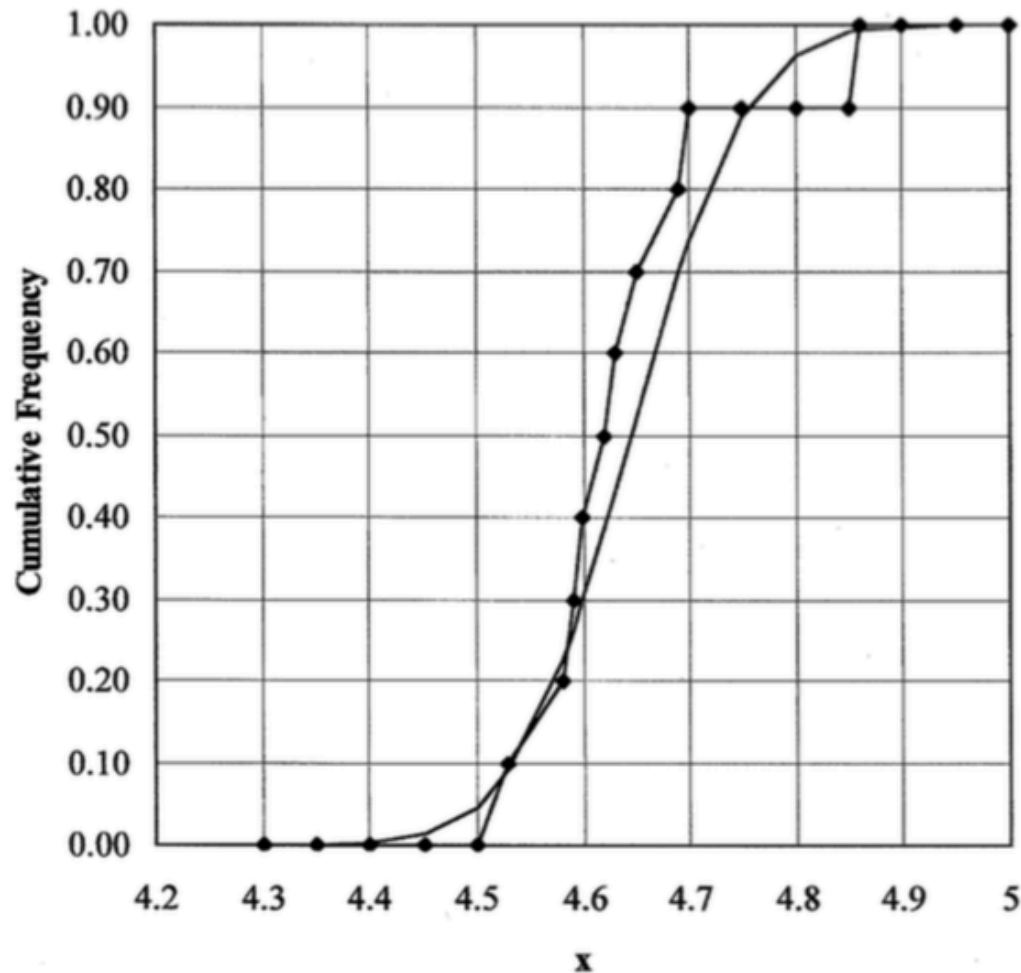
and Table 3.2, or a computer function. In this case the values were obtained with the probability functions in Microsoft Excel. Note the behavior of the normal distribution at large deviations from the mean value of x_m .

A graphical display of the actual and normal distribution frequencies is shown in example figure (b). The 50 percent value for the normal distribution occurs at $x = x_m = 4.645$. The actual frequency curve deviates substantially from the normal distribution in some regions of the chart.

A computer-generated comparison may be made by listing the values of x in ascending order as shown in the table below.

x	Actual Frequency	Normal Distribution Frequency
4.3	0.00	0.00003
4.35	0.00	0.00032
4.4	0.00	0.00229
4.45	0.00	0.01201
4.5	0.00	0.04665
4.53	0.10	0.09159
4.58	0.20	0.22593
4.59	0.30	0.26220
4.6	0.40	0.30124
4.62	0.50	0.38616
4.63	0.60	0.43109
4.65	0.70	0.52307
4.69	0.80	0.69876
4.7	0.90	0.73780
4.75	0.90	0.88787
4.8	0.90	0.96359
4.85	0.90	0.99117
4.86	1.00	0.99358
4.9	1.00	0.99842
4.95	1.00	0.99979
5	1.00	0.99998

Deney verilerinin grafiği



—●— Actual Frequency
— Normal Dist Frequency

Eğri Uydurmada Chi-kare testi

- Deneysel datalar için bir eğri uydurma yapıldığında, bu uydurulan eğri için ne kadar iyi olduğu belli değildir.
- Uydurmanın iyilik derecesi için Chi-Square testi yapılarak bu anlaşılabilir.

$$\chi^2 = \sum_{i=1}^n \frac{[(\text{observed value})_i - (\text{expected value})_i]^2}{(\text{expected value})_i}$$

burada n ile gözlem sayısı veya deney (hücre) sayısıdır.

- k ise beklenen dağılım
- Chi-squared. P olasılık değeri, verilen F serbestlik derecesi sayısının için tablodaki değeri

$$F = n - k$$

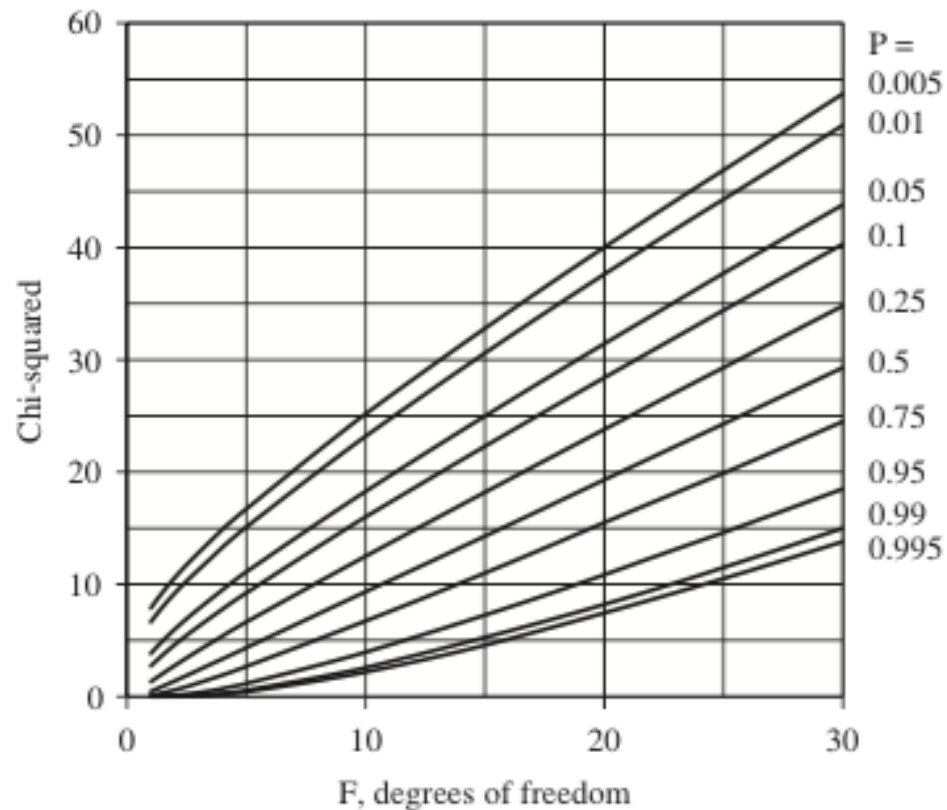


Table 3.6 Chi-squared. P is the probability that the value in the table will be exceeded for a given number of degrees of freedom F^\dagger

$P \backslash F =$	0.995	0.990	0.975	0.950	0.900	0.750	0.500	0.250	0.100	0.050	0.025	0.010	0.005
1	0.004393	0.007157	0.01062	0.01593	0.0198	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88
2	0.000100	0.00201	0.00506	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	0.0017	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	0.636	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.35	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.3	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

[†]From C. M. Thompson: *Biometrika*, vol. 32, 1941, as abridged by A. M. Mood and F. A. Graybill, *Introduction to the Theory of Statistics*, 2d ed., McGraw-Hill, New York, 1963.

Örnek

DEFECTS IN PLASTIC CUPS. A plastics company produces two types of styrofoam cups (call them A and B) which can experience eight kinds of defects. One hundred defective samples of each cup are collected and the number of each type of defect is determined. The following table results:

Type Defect	Cup A	Cup B
1	1	5
2	2	3
3	3	3
4	25	23
5	10	12
6	15	16
7	38	30
8	6	8
Total	100	100

We would like to know if the two cups have the same pattern of defects. To do this, we could compute chi-squared for cup B assuming cup A has the expected distribution. But we encounter a problem. Defects 1, 2, and 3 do not meet our criterion of a minimum of five expected values in each cell. So, we must reconstruct the cells by combining 1, 2, and 3 to obtain:

Type Defect	Cup A	Cup B
1, 2, 3	6	11
4	25	23
5	10	12
6	15	16
7	38	30
8	6	8
Total	100	100

For the former case we had eight cells or groups and one imposed condition (total observations = 100), so $F = 8 - 1 = 7$. After grouping defects 1, 2, and 3, we have $F = 6 - 1 = 5$. Using this new tabulation the value of chi-squared is calculated as 7.145. Consulting Table 3.6, we obtain the value of P as 0.43. Thus, we might expect that the two cups have approximately the same pattern of defects.

Uygulama-2

TOSS OF COIN: INFLUENCE OF ADDITIONAL DATA POINTS. A coin is tossed 20 times, resulting in 6 heads and 14 tails. Using the chi-square test, estimate the probability that the coin is unweighted. Suppose another set of tosses of the same coin is made and 8 heads and 12 tails are obtained. What is the probability of having an unweighted coin based on the information from both sets of data?

For each set of data we may make only two observations: the number of heads and the number of tails. Thus, $n = 2$. Furthermore, we impose one restriction on the data: the number of tosses is fixed. Thus, $k = 1$ and the number of degrees of freedom is

$$F = n - k = 2 - 1 = 1$$

The values of interest are:

	Observed	Expected
Heads	6	10
Tails	14	10

For these values χ^2 is calculated as

$$\chi^2 = \frac{(6 - 10)^2}{10} + \frac{(14 - 10)^2}{10} = 3.20$$

Consulting Table 3.6, we find $P = 0.078$; that is, there is an 8 percent chance that this distribution is just the result of random fluctuations and that the coin may be unweighted.

Now, consider the additional information we gain about the coin from the second set of observations. We then have four observations: the number of heads and tails in each set. There are only two restrictions on the data: the total number of tosses is fixed in each set. Thus, the number of degrees of freedom is

$$F = n - k = 4 - 2 = 2$$

For the second set of data the values of interest are:

	Observed	Expected
Heads	8	10
Tails	12	10

Chi-squared is now calculated on the basis of all four observations:

$$\chi^2 = \frac{(6 - 10)^2}{10} + \frac{(14 - 10)^2}{10} + \frac{(8 - 10)^2}{10} + \frac{(12 - 10)^2}{10} = 4.0$$

Consulting Table 3.6 again, we find $P = 0.15$.

En Küçük Kareler Metodu

$$S = \sum_{i=1}^n (x_i - x_m)^2$$

$$\frac{\partial S}{\partial x_m} = 0 = \sum_{i=1}^n -2(x_i - x_m) = -2 \left(\sum_{i=1}^n x_i - nx_m \right)$$

$$x_m = \frac{1}{n} \sum_{i=1}^n x_i$$

Uygulama-1

$$y = ax + b$$

$$S = \sum_{i=1}^n [y_i - (ax_i + b)]^2$$

$$\begin{aligned} nb + a \sum x_i &= \sum y_i \\ b \sum x_i + a \sum x_i^2 &= \sum x_i y_i \end{aligned}$$

$$a = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i y_i)(\sum x_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$$\hat{y} = ax + b$$

En Küçük Kareler -Uygulama

y_i	x_i
1.2	1.0
2.0	1.6
2.4	3.4
3.5	4.0
3.5	5.2
$\sum y_i = 12.6$	$\sum x_i = 15.2$

$$y = ax + b$$

KABUL EDİLEN DENKLEM

$$\frac{\partial S}{\partial x_m} = 0 = \sum_{i=1}^n -2(x_i - x_m) = -2 \left(\sum_{i=1}^n x_i - nx_m \right)$$

$$\begin{aligned} nb + a \sum x_i &= \sum y_i \\ b \sum x_i + a \sum x_i^2 &= \sum x_i y_i \end{aligned}$$

$$a = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$$b = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i y_i)(\sum x_i)}{n \sum x_i^2 - (\sum x_i)^2}$$

$x_i y_i$	x_i^2
1.2	1.0
3.2	2.56
8.16	11.56
14.0	16.0
18.2	27.04

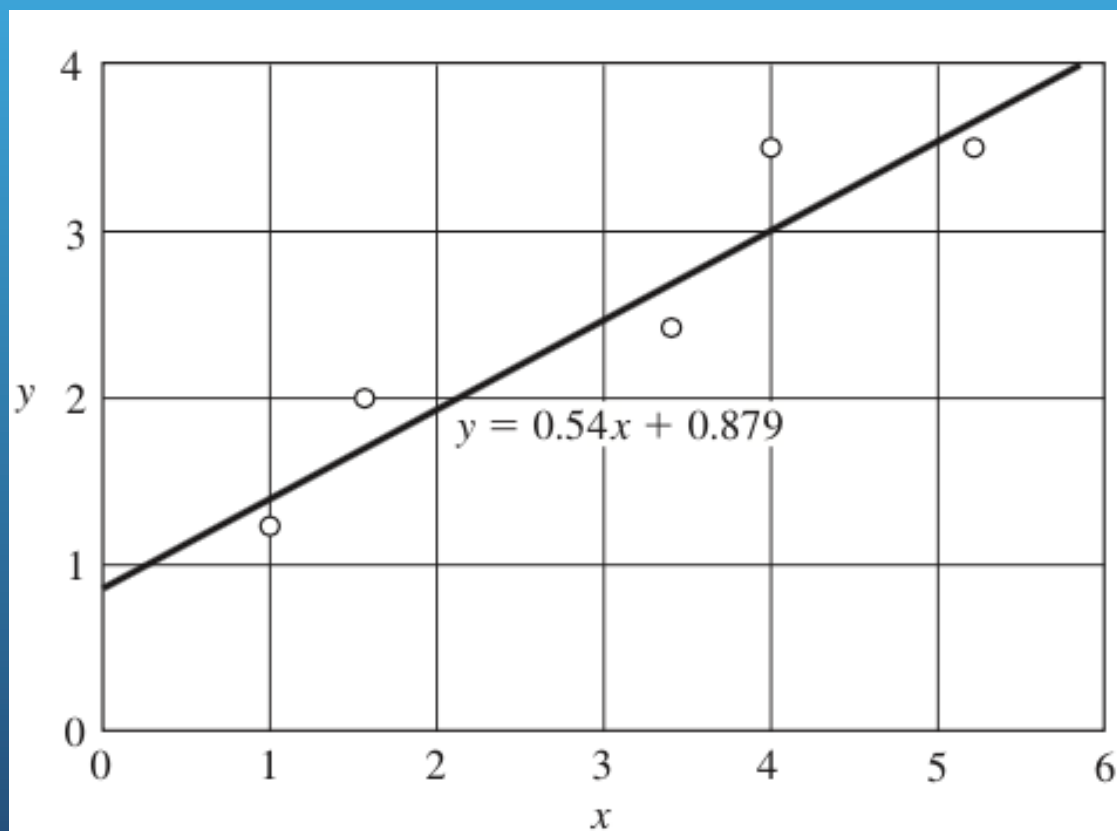
$$\sum x_i y_i = 44.76$$

$$\sum x_i^2 = 58.16$$

$$a = \frac{(5)(44.76) - (15.2)(12.6)}{(5)(58.16) - (15.2)^2} = 0.540$$

$$b = \frac{(12.6)(58.16) - (44.76)(15.2)}{(5)(58.16) - (15.2)^2} = 0.879$$

$$y = 0.540x + 0.879$$



KORELASYON SAYISI

$$r = \left[1 - \frac{\sigma_{y,x}^2}{\sigma_y^2} \right]^{1/2}$$

$$r^2 = \frac{\sigma_y^2 - \sigma_{y,x}^2}{\sigma_y^2}$$

$$\sigma_y = \left[\frac{\sum_{i=1}^n (y_i - y_m)^2}{n - 1} \right]^{1/2}$$

$$\sigma_{y,x} = \left[\frac{\sum_{i=1}^n (y_i - y_{ic})^2}{n - 2} \right]^{1/2}$$

$$y_m = \frac{\sum y_i}{n} = \frac{12.6}{5} = 2.52$$

$$y_{ic} = 0.5490x + 0.879$$

i	y_i	y_{ic}	$(y_i - y_{ic})^2$
1	1.2	1.419	0.048
2	2.0	1.743	0.066
3	2.4	2.715	0.0992
4	3.5	3.039	0.212
5	3.5	3.687	0.035
			$\sum = 0.4607$

$$\sigma_{y,x} = \left[\frac{\sum_{i=1}^n (y_i - y_{ic})^2}{n - 2} \right]^{1/2}$$

$$\sigma_{y,x} = \left(\frac{0.4607}{3} \right)^{1/2} = 0.3919$$

$$\sigma_y = \left[\frac{\sum_{i=1}^n (y_i - y_m)^2}{n - 1} \right]^{1/2}$$

$$y_m = (\Sigma y_i) / 5 = 2.52$$

$$\sigma_y = [\Sigma (y_i - y_m)^2 / (5 - 1)]^{1/2} = 0.987$$

$$r = \left[1 - \left(\frac{0.3919}{0.987} \right)^2 \right]^{1/2} = 0.9178$$